BASIC LANDSCAPING FORMULAS

For DIY or Contractor

ABSTRACT

Instructions on using the basic geometry formulas used for landscaping & hardscaping material calculations for various geometric shapes.

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Basic Landscaping Formulas

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Basic Landscaping Formulas

Important Notes

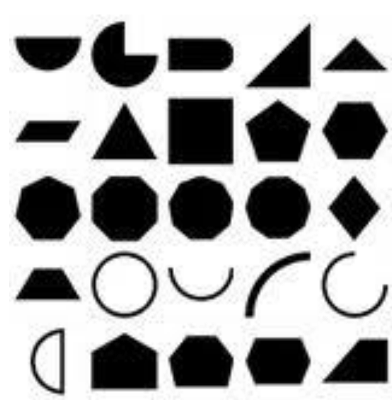
Many materials such as sands, soils, paver base and others can have a compaction factor of up to 30%. Most hard stone have minimal compaction, around 1% to 10%. You need to add to your results to allow for compaction and settling, especially if you plan to purposely compact it. So, you need to multiply your volume results by 1.01 to 1.3 (for 1% to 30%).

Example, paver base typically compacts 30%. So, if you need a 2-inch base (after compaction) you will need to order enough for 2.6 inches depth. *It is easier to calculate the total volume, such as cubic yards, then multiply by 1.3 to get required total.*

Accurate measuring is the most important part of the project. Inaccurate measurements will result in the wrong amount of material being purchased. As a professional landscaper you really should know this part well. Wrong quantities result in inaccurate bids that can lead to a loss if you're not careful.

The most common formulas you will ever use are the "area" and volume formulas. The formulas vary by the type of geometric shape you are dealing with. Modern houses may have multiple shapes of flower beds, walkways, etc. Note, for most calculations you will need to know the square feet or cubic feet (stone walls, etc.). Calculating the cubic yardage is determined by taking cubic feet and dividing by 27 (27 cu. ft. per cubic yard [i.e. 3'x3'x3'=27']). The L/S Material Calculator app allows you to enter the width, length and depth to do the calculations. Flower beds should be measured in feet for width and length. Depth should be measured in inches, unless noted otherwise. If you measured width and length in inches, simply divide by 12 to get value in feet. If you calculated area in inches such as 18 inches x 200 inches = 3600 inches, then divide the total square inches by 144 (3600 / 144 = 25 square feet). [144" = 12" x 12"] For example 18 inches = 1.5 Ft. Try not to round off decimals too much, perhaps to 4 digits.

The most common shape you'll run into is a simple rectangle. For example, a flower bed running down the side of a house. Here's an image of some common shapes you may also run into:



Squares, rectangles, triangles, ovals and circles are the easiest to calculate. You need to think in 2 dimensions to calculate the area (i.e. width and length). But, to fill a space, you need to think in 3 dimensions, width, length and depth. Each measurement needs to be the same measuring unit, such as inches or feet. Some shapes can be easier to calculate by breaking them into smaller squares, triangles, circles, etc. and then adding results together.

Quick calculations for the field:

For 2-inch depth: Take square feet and divide by 162 $\frac{Total\ Sq.Ft.}{162} = Cubic\ Yards$

For 2.5-inch depth: $\frac{Total\ Sq.Ft.}{129.6} = Cubic\ Yards$

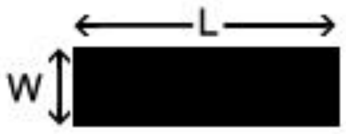
For 3-Inch depth: $\frac{Total\ Sq.Ft.}{108} = Cubic\ Yards$

For 4-inch depth: $\frac{Total\ Sq.Ft.}{81} = Cubic\ Yards$

Where did these 'magic' formulas come from? Want to create your own? Here's how to do it: Take 27 (i.e. 27 cubic feet per yard) and divide by the depth in Ft. To get depth in feet, take the depth in inches and divide by 12 (i.e. 12 inches per foot). Formula: $\frac{27\ Cubic\ Ft}{Depth\ (Ft.)} = Magic\ Divisor$

Rectangles/Squares

The basic square or rectangle (technically a square is a rectangle with equal sides). First, find the length and width (W and L):



$Area = W\ x\ L$

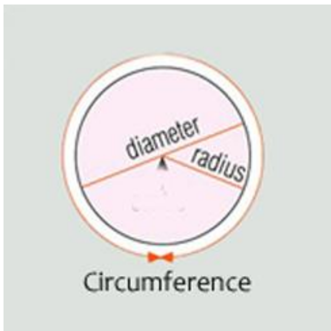
After finding the Area, multiply by the depth. Calculating cubic yards is as follows:

$$Cubic\ Yards = \frac{Area\ (in\ sq.ft.)\ x\ Depth\ (in\ ft.)}{27}$$

For example, if W=1.5 FT and L=30 FT and Depth = 2.5 Inches, Cubic Yards = 0.347

Circles

Next shape is a circle, very common for small trees and shrubs. Note, for a large tree or shrub you may want to subtract the area occupied by the tree or shrub to get a more accurate calculation. This simply requires doing three (3) calculations: one for the total area, one for the area of the tree or bush, and subtracting that from the total. (See Tree Ring Calculation.) Measuring diameter or radius of a large tree can be difficult.



The easiest way to do that is to measure the circumference around the tree (See image). A soft tape measure is best, like a tailor's measuring tape. Or use a string then measure the string afterwards. To get the diameter from the circumference, take circumference and divide by pi (3.141592654) $\frac{Circumference\ (Ft.)}{\pi} = Diameter\ (Ft.)$ and the radius = diameter divided by 2.

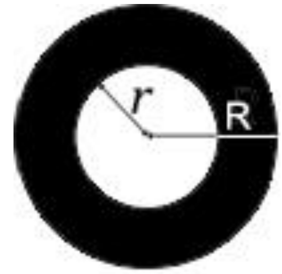
The basic circle area calculation, without considering a tree or shrub is like this:

First, find the diameter of the circle (measure straight across the widest portion), then divide by 2 to get the radius *r* for the formula: $Area = \pi r^2$ This means Area = pi (3.141592654) x radius x radius

For example, a circle with a 10 Ft. diameter (radius = 5 ft): Area = 3.141592654 x 5 x 5

Which is equal to 78.5398 Sq. Ft. = 0.606 cubic yards at 2.5 inch depth ($\frac{78.5398}{129.6}$)

Ring/Firepit



Doing a 'ring' calculation, a very common use, (for subtracting inner volume, or for calculating a circular stone wall with something else in the center, like a flower bed, or fire pit) and finding remaining area: Use the hints mentioned above to find the radius of both circles. R = radius of outer circle from center, r = radius of inner circle (from same center point). There's 2 ways of calculating, the hard way and easy way.

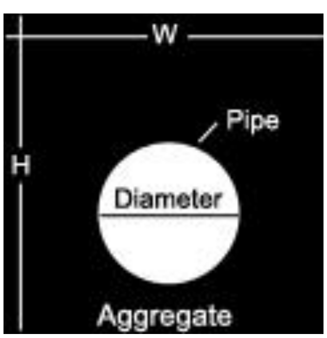
The easy way is like this: $Area = \pi\ x\ (R^2 - r^2)$ Meaning take (inner radius x inner radius) [r x r] and subtract from (outer radius x outer radius) [R x R] then multiply by pi (3.141592654). You should use feet for measurements, or convert from inches afterward: $\frac{Square\ Inches}{144} = Square\ Feet$

The resulting Area value is the area of the outer ring, excluding the inner circle. The hard way to calculate is to find the area of the outer circle, then subtract from that, the area of the inner circle. To calculate cubic feet of outer ring, for a stone wall, for example, multiply the Area (in feet) by the depth in feet. $Area\ (Sq.Ft.) x\ Depth\ (Ft.) = Cubic\ Feet$ then divide by 15± = Tons needed. Or divide by 27 = cubic yards needed. *(Note, we use 15 cubic ft. per ton as a standard. However, this number depends on the density of the stone, which can vary from 13 to 20 cu. Ft. per ton.)*

Use the same formula to calculate cubic feet of the inner circle, if you need to fill it up with dirt, for example. $Cubic\ Feet = Area\ (inner\ circle\ Sq.Ft.)\ x\ Depth\ (Ft.)$ This is also known as a 'cylinder' calculation.

Drainage Ditch & Pipe

A cylinder calculation might be useful when doing drainage pipe, to subtract the volume the pipe occupies from the total volume of the ditch you put it in.



First calculate the volume of the ditch:

$$Volume\ (Cubic\ Ft.) = Width\ (Ft.)\ x\ Height(Ft.)\ x\ Length(Ft.)$$

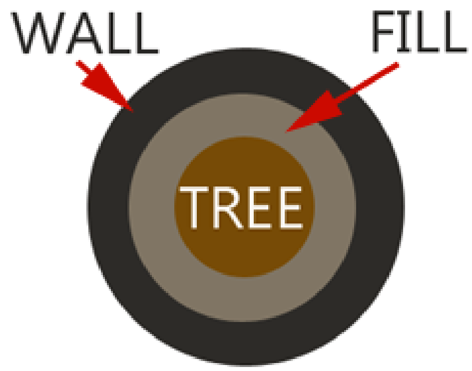
Now calculate the volume of the pipe (radius = diameter/2):

$$Pipe\ End\ Area = \pi r^2$$

$$Pipe\ Volume\ (Cubic\ Ft.) = Pipe\ End\ Area\ x\ Pipe\ Length\ (Ft.)$$

Now, simply subtract pipe volume from ditch volume to get ditch aggregate needed. Note, for a small ditch and short pipe, it's probably not worth the effort to calculate. For a long ditch, or a big pipe, like a 3-foot diameter pipe, for example, you would want to.

Tree Ring



A tree ring is a double ring and is a slightly different calculation than just a ring. You will have a tree in the center, a ring around the tree and an outer ring for the wall. The ring around the tree will be filled with dirt, mulch or stone.

The first step is to measure the circumference of the tree. From that we can calculate the diameter and radius of it (diameter / 2). You will essentially be doing two separate ring calculations. Once you have the tree's radius, add to it the width of the bed (fill area) you want. For example, if you want it to be 2 ft. wide all around the tree, then the radius for that ring is the tree radius + 2 ft. We will use the same formula as the ring calculation. In this formula, r = tree radius, R = bed radius.

$$Bed\ Area = \pi\ x\ (R^2 - r^2)$$

Let's say the tree radius = 1 ft and your desired bed width is 2 ft. and you want the bed (fill) 1 ft. deep.

Bed Area = π x ((3 x 3) – (1 x 1)) = 3.14159 x (9 - 1) = 25.133 Sq. Ft.

Bed Cubic Ft. = 25.133 (@ 1 ft deep) and Cubic yards (CYD) of fill = 25.133 / 27 = 0.93 CYD.

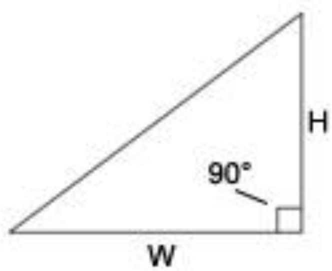
Now, to calculate the wall volume, we take the bed radius and add the width of the wall to it, to get the outer radius. As an example the wall be 1 ft. deep (to match the bed) and 1 ft. wide. Remember the bed radius is 3 ft., so the wall radius is 4 ft. Plug that into the ring equation:

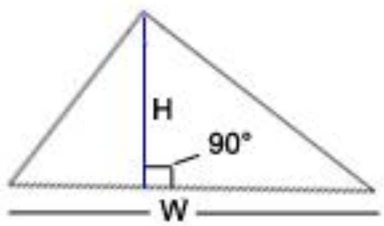
Wall area = π ((4 x 4) – (3 x 3)) = 3.14159 x (16 – 9) = 21.99 Sq. Ft.

Wall Cubic Ft. = 21.99 and amount of stone = 21.99 / 15 = 1.466± Tons

Triangular

Triangles are also very simple to calculate. It does not matter what the triangle looks like really, the same formula is used for various triangular configurations. Basically, a triangle calculation is the same as a square/rectangle, but cut in half. Looking at the 2nd image below, in reality you'd be calculating 2 rectangles, cut in half. But, both shown below use the same formula.

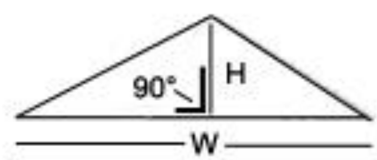




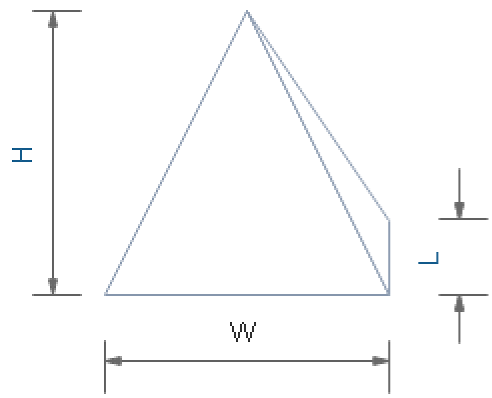
$$Area(Sq.Ft.) = \frac{Width\ (Ft.)\ x\ Height(Ft.)}{2}$$

It's important to note, though, when measuring the height, run a line from the tip point (apex) to a point that is 90° in line with the width line (opposite from the tip). Use a framing square, if necessary. The more accurate you do that, the more accurate your results.

In case you were not aware, both of the above triangles are identicle, because the tip of the points are both opposite 90° angles. Off angles do not matter providing you measure the tip point to a point 90° off the opposite side (usually the widest side).



Pyramid



1 Photo courtesy US Air Force[1]

A pyramid shape is probably not in high demand in landscaping. But, you should know how to calculate one, just in case someone wants a big stone pyramid. You will be calculating volume in cubic feet for a pyramid.

The basic formula for a pyramid with a square or rectangular base is:

$$\frac{L\ x\ W\ x\ H}{3} = Volume$$

It can also be translated to: base area [L x W] (sq. ft.) x height (ft.) / 3)

$$\frac{A_{b\ x\ H}}{3} = Volume$$

Where A_b is the area of the base (L x W) [in ft.].

Example:

The Khufu pyramid at Giza has a W & L of 756 Ft. and a height of 455 Ft. (current after erosion). (Wikipedia contributors, 2018)[1]

So, plug that into the equation:

$$\frac{756\ x\ 756\ x\ 455}{3} = 86{,}682{,}960\ Cubic\ Ft.$$

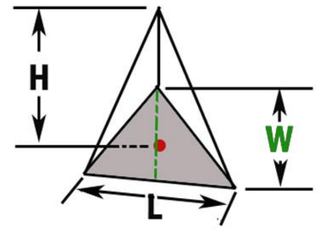
Figuring with the 15-cu. ft. per ton method, that is 5,778,864 Tons. (Wow, at about $200/ton it would cost you $ 1,155,772,800 [$1.16 Billion]).

A more practical application would be a pyramid monument, say 4 ft. tall. Let’s calculate one like that.

An example monument: using a height of 4 ft. with a width and length of 4 ft.

$$\frac{4\ x\ 4\ x\ 4}{3} = 21.333\ Cubic\ Ft.or\ 1.422\ Tons$$

If the pyramid has a triangular base instead of rectangular/square, the calculation is similar. Figure the base area like the triangle formula above. Where W & L are width and length in ft. A_b is the base area in sq. ft. *Remember the W is from the apex opposite the L side, to a point that is 90 degrees off the L side.*



$$A_b = \frac{W \ x \ L}{2}$$

Then multiply by the height, then divide by 3.

$$\frac{A_{b \ x \ H}}{3} = Volume$$

Volume = Cubic Ft., so $\frac{Volume}{15} = Tons\ needed$

Mounds

A mound is another common calculation. Most landscapers will mound a flower bed to allow for settling over time (like a cemetery grave). This type of mound calculation is like calculating the partial volume of a tank (such as a cylindrical fuel tank). The formula is complex and is a multi-step process. To simplify calculations, we will use a spherical cap, split in half, for each end of the bed and a standard (partial) cylinder for the long section.

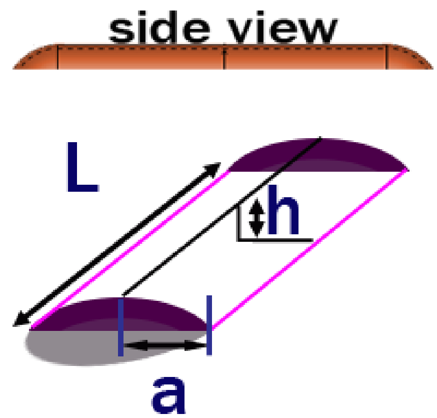


Figure 2 - Elongated Mound

In Fig. 2, we need the width from center to edge [a] or take full width [c] (see Fig.3) and divide by 2, the height from the ground plane to the apex [h] (top or highest point) and the length [L] (excluding the rounded ends). The formula needs the radius, which we will calculate from the value [a] and value [h]. *Write down each calculation result, you will need them later.*

Note: a = c / 2 or c = 2 x a and the end pieces widths = a.

The formula for the Radius [R]:

$$R = \frac{h}{2} + \frac{(2a)^2}{8h}$$

Next, we need to find the angle.

Where θ = angle. We will use angle in radians (not degrees).

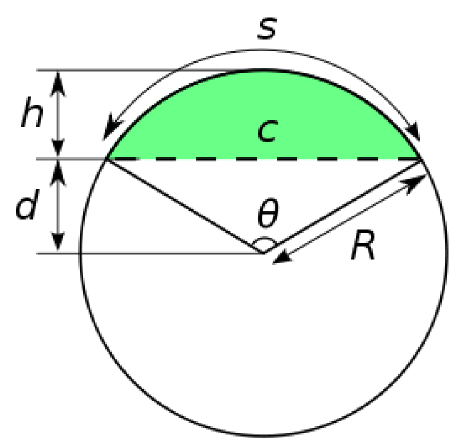


Figure 3 - Segmented circle[2]

Where $cos^{-1} = arccos$. You will need a scientific calculator that supports this, to do these calculations.

$$\theta = 2\cos^{-1}(1 - \frac{h}{R})$$

The arc length $s = \theta \, x \, R$. Write this down. *You will need it if you want to calculate surface area.* The formula for the end piece area [A] (partial [segmented] circle [purple sections in fig. 2, green in fig. 3]):

$$A = \frac{1}{2} r^2 (\theta - \sin\,\theta)$$

Where θ = angle in radians (not degrees). The area A = angle minus the sin of the angle times radius squared (r times r) times 0.5

Now, multiply A by the length [L in fig. 2] to get the volume of the main portion. The next step is to add the rounded ends (split spherical cap). You can add the total spherical cap volume to the main volume because half goes on either end. You will use the same measurements for the cap.

Spherical Cap Volume

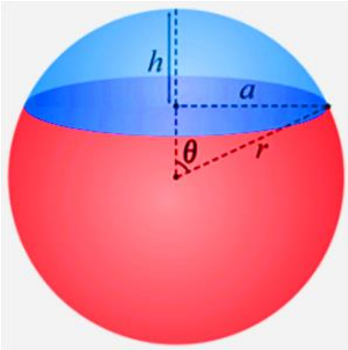


Figure 4 - Image courtesy Wikipedia[3]

The spherical cap volume formula:

$$V = \frac{\pi h^2}{3}(3R - h)$$

Where h is the height and R is Radius (from previous calculation above). You can use 3.14159 for π and calculate like this:

First, (h x h) x 3.14159 / 3 then multiply by ((3 x R) – h) which will give you the volume [V] in cubic ft. Then add that to the volume of the main mound section to get total cubic ft., then divide by 27 to get cubic yards.

If you need to know the surface area of the mound(s), for example to cover with sod (grass), mulch or stone, use the following formulas:

Spherical cap surface area (two ends combined):

$$Cap\ Surface\ Area = 2\pi r h$$

Elongated mound surface area:

$$Mound\ Area(s\ x\ L)$$

Where s is the arc length calculated above and L is the length. Add the *Cap Surface Area* to *Mound Area* to get total surface area.

Because these calculations are time consuming, you should consider using my L/S Material Calculator app for Android (available in the Google Play store) for faster field calculations.

Arcs & Arches

An arc or arch is called a circle segment (partial circle) and is smaller than a half circle. To calculate an arch's sq. ft. is fairly complex. There are three formulas to use and two separate calculations have to be done. A scientific calculator is needed to complete it. These calculations are usually for an arced/curved driveway.

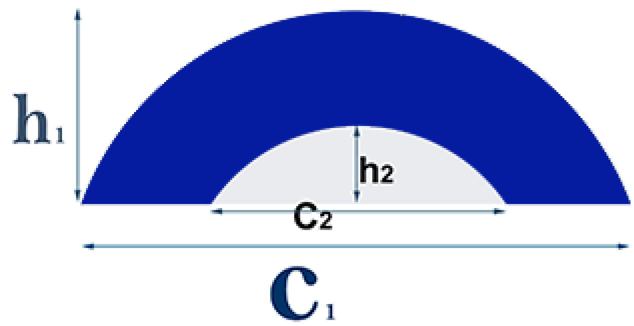


Figure 5 - Driveway Arc

First, you need to measure the total width and height (h_1 and C_1 in Fig. 5). Calculate that sq. ft. Then, get the h_2 and C_2 and calculate that sq. ft. Subtract the h_2-C_2 sq. ft. from the h_1-C_1 sq. ft. to get the sq. ft. for the arch area. (Blue shaded area).

The first calculation will be to find the Radius of the entire segment using the h_1 and C_1 values for h and C.

$$R = \frac{h}{2} + \frac{C^2}{8h}$$

Then we need to find the angle (angle from the wide ends of C to the center of the larger *hypothetical* circle).

$$\text{Angle } \theta = 2\cos^{-1}(1 - \frac{h}{R})$$

Then calculate the area:

$$A_1 = \frac{1}{2}R^2(\theta - \sin\ \theta)$$

Now, we repeat for the inside arc using h_2 and C_2 for the h and C values.

$$R = \frac{h}{2} + \frac{C^2}{8h}$$

$$\text{Angle } \theta = 2\cos^{-1}(1 - \frac{h}{R})$$

$$A_2 = \frac{1}{2}R^2(\theta - \sin\ \theta)$$

Driveway section (segment) Sq. Ft. = $A_1 - A_2$

NOTES:

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